# Wideband Multiuser OFDM: Implementation and Capacity

**Caleb Kemere**
**Stanford University**
**June, 2001**

## Abstract

Optimal bit and power allocation in OFDM wireless systems allows for dramatic increases in user and data capacity. This paper focuses on two main fronts. First, previous results are extended to include time-varying fading. Furthermore, a simple (but novel, or at least, not previously published) optimization goal, maximizing the sum of user rates, and an iterative algorithm is presented which allows for pseudo-optimal results. These are compared with previous work using a time-varying correlated Rayleigh frequency selective channel model. Second, the problem of high peak-to-average signal ratio is widely understood and studied within the OFDM community. However, attention has not been given to what effect optimal power and bit allocation have on the problem. Some simulation results are presented, and a direction for analytical work is proposed.

## Model

High speed wireless data is the current industry fad. While there have not been successful wide-area implementations of this paradigm, many are hopeful that consumers desire access to large bandwidth data in locations in which it is inconvenient for them to be plugged in to a wall. As a result, much of the research in the last few years has centered on technologies that would enable these high bandwidth systems. While some researchers are focusing on *ad hoc* network architectures, it is likely that structured networks will form the backbone of any high speed wireless infrastructure for the next decade. As a result, this paper centers on systems structured in such a way that there is one node (the base station) that is communicating with the rest of the nodes in a time-division-duplexing paradigm, and (somehow) is able to develop perfect information regarding the channels between it and the users.

The attenuation of a high frequency wireless channels along a particular, (speficically not direct "line-of-sight") path is well modeled in magnitude (given standard assumptions (1)) by a Rayleigh random variable. In this study, a frequency selective fading channel is constructed by forming an exponentially decreasing impulse response, where each of the five taps are Rayleigh random variates with uniformly distributed random phases. Within a symbol time, the channels between each user and the base station are assumed to be constant. However, to further enhance the channel model, between symbol times, each tap follows an independent, but self-correlated path according to the an auto-correlation function modeled on slow motion assuming a unidirectional received beam (see (2)). Figure 1 shows a time-segment of the channel. The correlated Rayleigh variables are generated according to (2), with an FFT size of 1024 (thus, 1024 are generated simultaneously. Scaling ensures that the average subcarrier power will be unity. Additionally, the power of the noise in each subcarrier band is assumed to be unity (without loss of generality). One failure of this model is that it only includes the fading process, and ignores path loss and shadowing. These omissions are especially unfortunate for the peak-power and clipping simulations. However, the assumption is made that the power amplifiers can be controlled using a feedback mechanism to compensate for the (relatively slow) processes of shadowing and path loss. In fact, the maximum and minimum signal amplitudes of a power amplifier can indeed be adjusted by adjusting the maximum and minimum voltage sources in the circuit. This adjustment process, however, is very inexact compared to DSP processing (8).

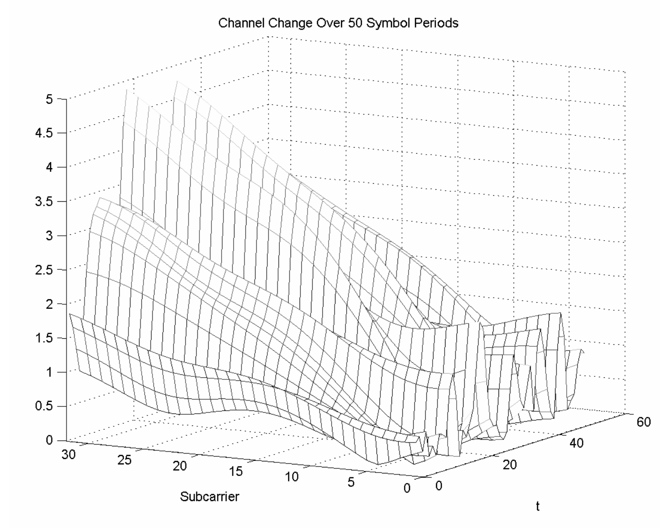


**Figure 1: Channel Model**

Since the seminal paper ((3)) more than 20 years ago, it has been recognized that for capacity-maximizing transmission (in a multi-user total data rate sense), it is optimal to "waterfill" in a communications channel. The basic intuition involved is to transmit a lot of data and power in good channel states, and less in bad ones. Thus, in a frequency selective fading the optimal power is frequency dependent. The channel states can be orthogonally decomposed into (simpler) non-frequency selective fading subcarriers using a discrete Fourier transform (DFT). Thus, modulation schemes which involve the DFT (such as OFDM) are naturally suited for optimal bit and power allocation.

**Adaptive OFDM**

As a result of the linear nature of the DFT, it is not difficult to define optimality criteria, and form the goal of maximizing capacity as a convex optimization problem. In systems with a single user, adaptive waterfilling techniques are well developed, and are used in many systems, most notably various generations of DSL. As the cusp of technology has migrated from wired (e.g., DSL systems) to wireless ones, interest in multiple user systems has grown. In several papers ((4), (5), (6)) the key result is that allowing multiple users to share a particular time evolution of a channel results in dramatic performance increases. This makes intuitive sense, while it is not unlikely that a given user will at a given time have large attenuations in some of her subcarriers, it is unlikely that several users poor subcarriers (assuming fast fading as the main channel impediment) will intersect.

When approaching the problem of how to optimally allocate power and bits to OFDM subcarriers, it is somewhat difficult to establish what metric is best. In (4) and (6), the optimal allocation scheme results in constant bit-rate transmission for each user, and minimum overall transmit power. In (5), the optimal allocation gives maximum data rate subject to a total transmit power constraint. In terms of system design, both power and rate are important. One might argue for power to be the key metric, given it's importance for calculating the size of cells and in determining other system parameters. However, it seems that rate is an equally good, if not better metric, here chosen especially for its close relation to system capacity.

Given that one wishes to maximize data rates subject to a transmit power constraint, the problem must be further defined to specify how the rates of multiple users are compared with each other. The obvious metric (and the one for which a simple iterative algorithm is presented), is to maximize the sum of all users rates (similar to a 1-norm). Alternatives exist, however. For example, certain users could be given priorities, or others could have a specified (achievable) minimum data rate. One particularly interesting metric, presented in (5), is to maximize the minimum rate of all the users. A comparison will be made in the following section between these two metrics.

**New additions**

Two issues must be considered, before the optimization process can be stated. First, the power constraint is different in single transmitter (broadcast channel) or multiple transmitter (multiple access channel) situations. The multiple access channel is more difficult because the power constraint becomes less directly correlated with the optimal channel allocation (more on this later). The second issue to be resolved is the rate metric of the optimization. As discussed, here, the maximum total data rate, or 1-norm, was chosen. The problem with a 1-norm is defined this way:

$$\text{maximize} \quad \sum_{k=1}^{K} |R_k|$$

$$\text{subject to} \quad R_k = \sum_{n=1}^{N} c_{nk}$$

$$\sum_{k=1}^{K-1} \sum_{n=0}^{N-1} f(c_{nk}) < P_{\max}$$

$$(c_{nk} \neq 1) \Rightarrow (c_{nj} = 0) \text{ for all } j \neq k$$

$$c_{nk} \in \{0, 1, 2, \ldots\}$$

The third constraint, representing the fact requirement that users not share channels and the fourth, the discrete domain of the $c_{nk}$'s (number of bits per subcarrier), make the problem non-convex. Typically, one would proceed by relaxing both of these constraints, and then solving the convex relaxed problem. Comparisons of the results of such analyses in (4) and (5) suggest that discrete solutions, such as the following, produce results that differ little from the optimal solution of the relaxed problem. In fact, it is not unlikely that for a single user, the solution given produces an optimal solution. This algorithm consists of simple water filling (greedy bit allocation), in which the best users in each subcarrier are given that subcarrier to occupy (consider the similarity to the single user case, see discussion in (4)).

**Greedy Optimization**

```
dP = 0
(n,k) = min(dP)
while (( P_total + dP_nk) < P_max)
  c_nk++
  dP_nk = (P_req(c_nk+1) – P_req (c_nk)) / H_nk
  for (j ≠ k) dP_nj = ∞
  (n,k) = min(dP)
end
```

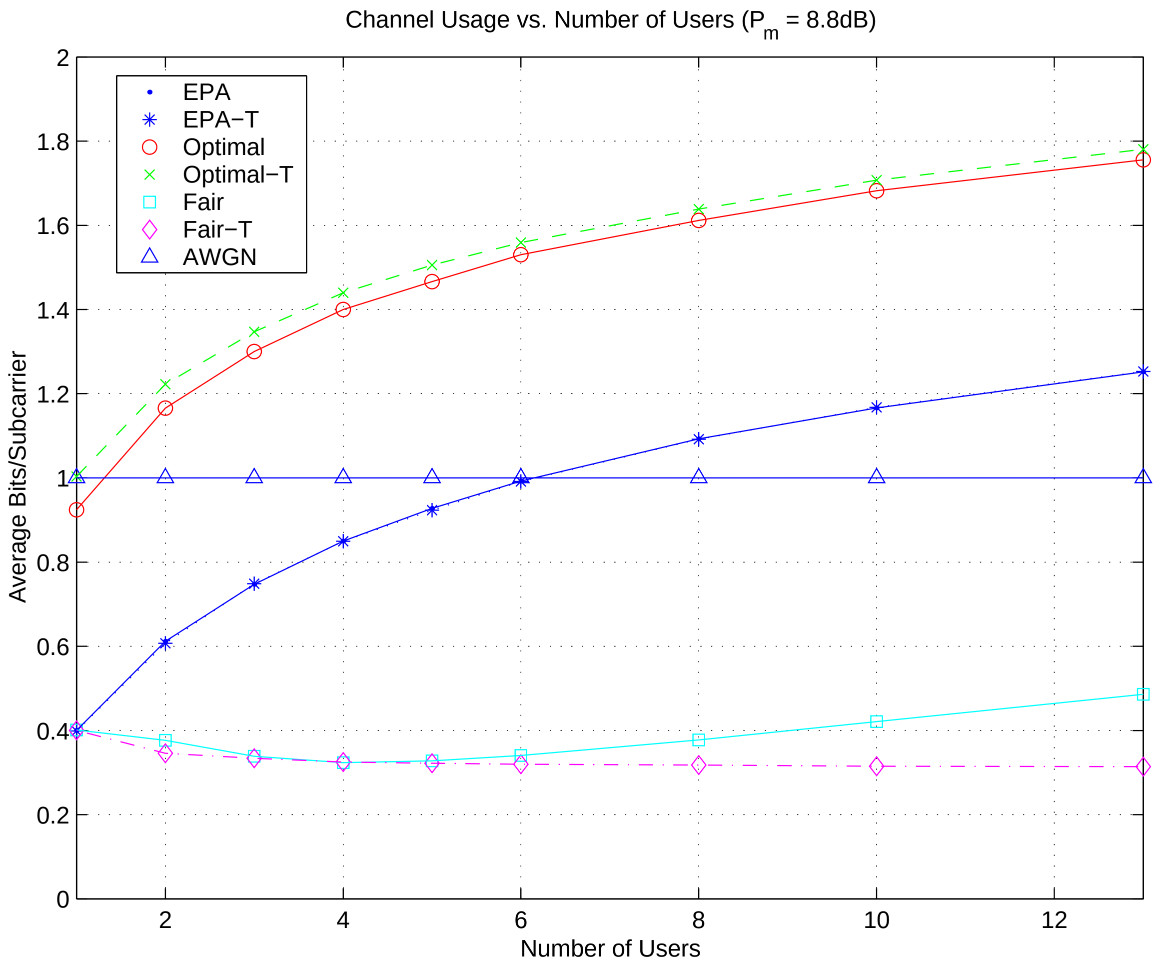


**Figure 2: Channel Usage of Optimal Schemes**

One of the contraindications for this optimization is that because it is greedy, it tends to be unfair to users with poor channel states. Notice also that it does not easily convert to a multiple

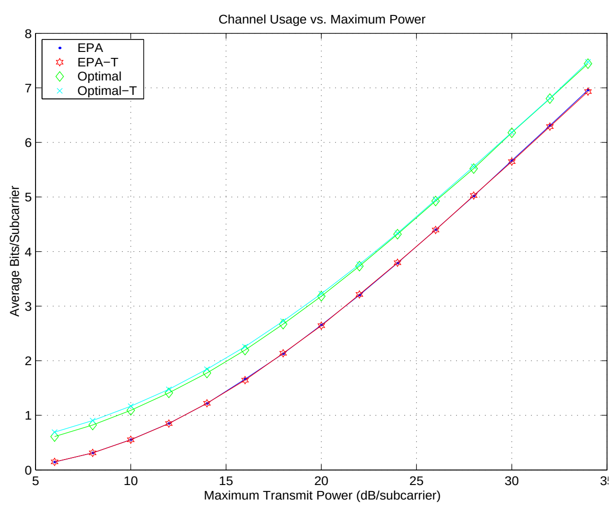


**Figure 3: Channel Usage vs. Transmit Power**

access situation. There, the problem is that one user could be allocated several subcarriers, but then only occupy them to the depth of one bit, whereas she could have used the same power to transmit slightly fewer bits in a single subcarrier, and other users with only slightly worse channels could then use the released subcarriers, resulting in a larger total supportable data rate.

In addition to optimization algorithm presented, a modification of the one found in (5) was considered. The major simplification for Rhee's algorithm is that power is not allocated differently (optimally) between subcarriers. Rather, a given subcarrier is assigned the best modulation scheme that it can support, given equal power allocated to each subcarrier. This modification drastically simplifies analysis of the algorithm, but, as shown in Figure 2, it results in a loss of performance.

Figure 2 is a comparison, in the single user case, of equal power and fully optimal schemes. (Note that the x–axis is labeled as the maximum transmit power per subcarrier, though it is only allocated that way for the EBA cases). There is a persistent penalty of about 0.5 bits/subcarrier for equal power allocation, though it decreases with increasing transmit power. One of the benefits of generating a time-correlated channel model is that it allows for further analysis of the optimal algorithms. The previously presented algorithms optimize the user's bit and power across frequencies. Adding another dimension, time, extends the problem, and seems to be an obvious method for improvement. In this scenario, one would not only optimally allocate resources across frequencies, but also across time as the channel changed (slowly) from symbol to symbol. However, as shown in Figure 2, while extending the channel vector into the time dimension has some benefit, it decreases with increasing transmit power, and becomes negligible.

However, allowing multiple users to utilize the channel at the same time results in quite a significant performance improvement. Figure 3 depicts the increase in capacity (average bits / OFDM symbol subcarrier) as the number of users increases. Notice that optimal allocation performs significantly better than equal-power. Recalling the discussion of optimality conditions, a comparison was made with the fair (max-min) optimization scheme from (5). To summarize it, one subcarrier is initially allocated to each user, who's data rate is then calculated based on an equal-power scheme. Following, the remaining subcarriers are iteratively allocated to the users with the current minimum total data rate. Some modifications were made to algorithm presented. Namely, rather than initially assigning subcarriers to users in (arbitrary) order of user number, the initial subcarriers are assigned in order of performance. The results of the fair allocation schemes seem rather poor, but it is important to notice that the "optimal" scheme will, in general give an average of 3-4 users the entire channel at one time. The fair scheme, however, is forced to ensure that all users are allowed use. It is important to note that as part of the algorithm for user allocation, if a user cannot (with the equally allocated power) transmit in any of the available channels, they are dropped from the pool. Thus, the increasing channel usage shown by the fair allocation scheme in Figure 3 may be an artifact of the number of users approaching the number of available subcarriers (31 in this case), and thus more easily being eliminated from the user pool. Figure 4 plots the average usage (number of users) for optimal, equal-power, and fair schemes. The normal fair allocation scheme is seen to decrease from full occupation as the number of users increase, supporting the earlier suggestion. However, the most interesting curve is actually the fair scheme where the channel is extended over time. Here,

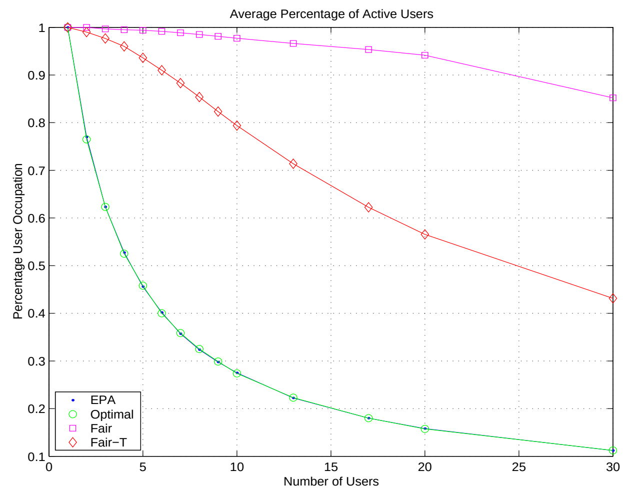


**Figure 4: Average Channel Occupation ($P_M$=17dB)**

we see that the user occupation drops significantly. What this implies is that it may be possible to develop a time-division multiple access scheme where only 3-4 users would be assigned the channel at any one time, and still achieve near-optimal results.

One extension of these results would quickly test this hypothesis. Additionally, one of the reasons that the fair optimization performs poorly is that it is usually able to assign subcarriers to each user. The rate-sum maximization, on the other hand, does not typically assign subcarriers to each user. A clear next step would be to develop a "pseudo-fair" allocation scheme with a well designed cutoff-channel state, which would allow users with generally poor channels to be eliminated from the current assignment. Additionally, the current results were not sensitive to the Doppler frequency, and it would be interesting to explore reasons why this might be.

### Adaptive OFDM and PAR

One of the major impediments of OFDM systems is that this type of modulation is prone to having a high peak-to-average signal ratio (PAR). Because OFDM decomposes a wideband signal into a series of narrow band ones, with low probability, the narrow band signals will add coherently and cause a large peak value. In general, PAR can exceed (in the case of high dimension constellations) N, where N is the number of carriers. High PAR has an impact on the system design mainly at the power amplifier. To ensure orthogonality of carriers, the amplifiers used in OFDM systems must be very linear. Thus, class A designs are nearly always used. To ensure linear transmission of all signals, the linear region of the transistors has to include both the maximum peak signal and the smaller average. Because class-A amplifiers drain DC current proportionally to their maximum and minimum limits (a constant loss irrespective of actual amplification), setting these limits wider has a multiplicative effect on reduction in power efficiency.

The problem of PAR is well characterized for the single user situations. Tellado's thesis (7) not only gives a good introduction to the problem, but also addresses several mitigation techniques. Briefly, typical methods for reducing PAR include tone injection/reservation, increasing constellation sizes and adaptively picking points out of the larger constellations, and MMSE post-detection of a non-linearly clipped signal.

Despite the various mitigation techniques, most systems are designed so that if the lowest probability high peak signals occur, they are allowed to saturate the amplifier, or are clipped digitally before it. Thus, it is useful and interesting to characterize the peak power performance of the various optimal OFDM bit and power allocations. Figure 5 shows the peak power distribution of four types of power allocation compared to a typical (non-optimized) OFDM symbol. Notice that, while the optimal bit allocation scheme has, in general, lower peak power, the equal-power allocations result in a slightly better PAR. Also, it is interesting to notice the change in the shape of the distribution of the optimal allocation scheme from allocation just in one symbol to allocating across time. The distribution becomes significantly wider, implying slightly worse performance (i.e., increased probability of clipping). Fundamentally important, although reasonable, is the recognition that optimal adaptation to the channel state worsens the PAR situation.

A further area of research is to correctly characterize the peak power characteristics of the optimal allocation schemes. While they appear Gaussian in form, a further analysis would characterize them analytically. Given an analytic representation for the peak power distribution, it would be possible to develop a model of the clipping noise process, and then consider optimal allocation schemes with appropriate constraints to limit clipping. Iterative techniques are not well suited to the problem. One could imagine choosing an optimal bit and power allocation, then testing it for clipping, and iteratively removing bits until the peak power satisfies some constraint. Unfortunately, signal peak power depends largely on the exact data being transmitted. Hence, the proposed algorithm, which already is exponential, would have to be done for each transmitted symbol. Alternatively, one could assume that clipped signals would simply add noise to the system, and analyze the BER assuming some sort of

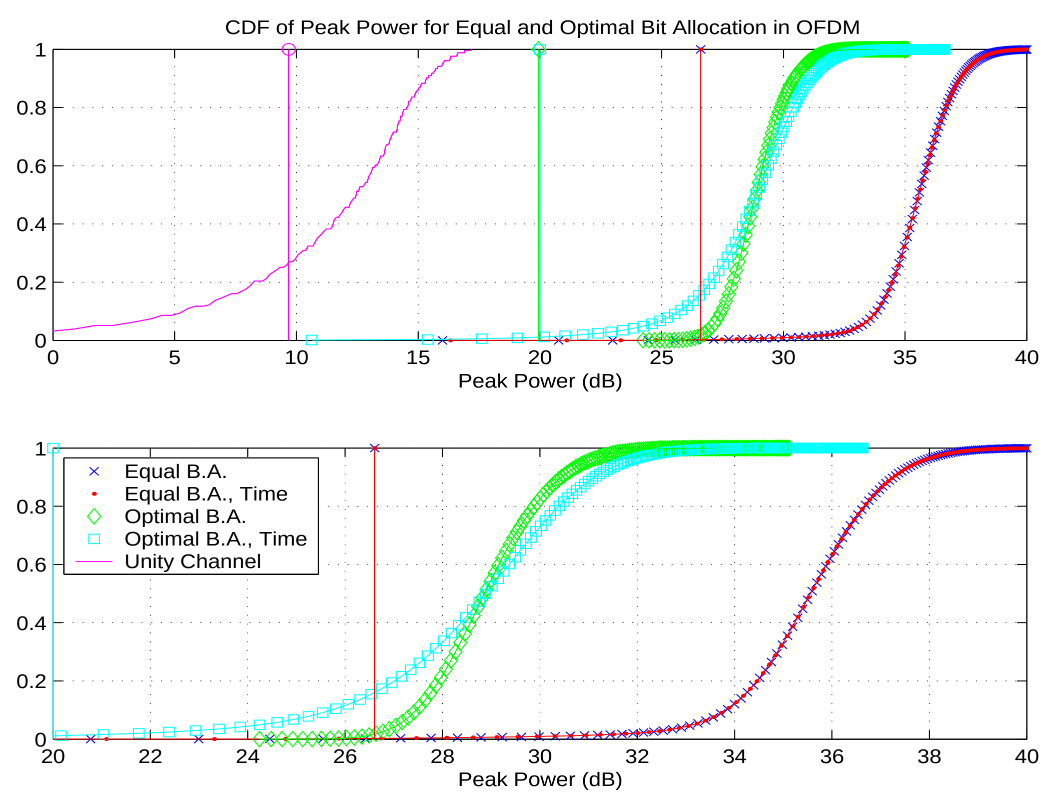


**Figure 5: Peak Power Distributions ($P_m$ = 20dB)**

coding scheme to recover bits lost to clipping noise. Either way, further work is necessary

**Conclusion**

As the demand for high speed wireless data modulation schemes increases, the desire to efficiently utilize spectral resources will become more apparent. In light of this, optimal resource allocation will become more and more necessary. Using time as an addition resource to frequencies and users appears to be a useful tool, but one which is limited. An additional degree of freedom does not add much utility to the system. However, a multiuser scheme in which sets of users would transmit with some fairness constraint over an optimal subset of channel states seems to be useful. The results of the fairness-over-time algorithm support the possibility of such a scheme.

The effect of clipping noise on capacity remains a pertinent issue that definitely also deserves more consideration.

**References**


1. T. Rappaport, *Wireless Communications*, Upper Saddle River, NJ, Prentice Hall, 1996.
2. D. Young and N. Beaulieu, "The Generation of Correlated Rayleigh Random Variates by Inverse Discrete Fourier Transform," *IEEE Trans. Comm.*, vol. 48, pp. 1114-1127, July 200.
3. T.M. Cover, "Broadcast Channels," *IEEE Trans. Info. Theory*, vol. IT-18, pp. 2-14, January 1972.
4. C. Wong, R. Cheng, K. Letaief, and R. Murch, "Multiuser OFDM with Adaptive Subcarrier, Bit, and Power Allocation," *IEEE JSAC*, vol. 17, pp. 1747-1758, October 1999.
5. W. Rhee and J. Cioffi, "Increase in Capacity of Multiuser OFDM System Using Dynamic Subchannel Allocation," *Proc. VTC-2000*, vol. 2, pp. 1085-1089, 2000.
6. H. Yin and H. Liu, "An Efficient Multiuser Loading Algorithm for OFDM-based Broadband Wireless Systems," *Globecomm 2000*, vol. 1, pp. 103-107, 2000.
7. J. Tellado-Mourelo, *Peak to Average Power Reduction for Multicarrier Modulation*, PhD Thesis, Stanford University, September 1999.
8. T. Meng, Personal Communication.