Channel Estimation in Wideband Multiuser OFDM Using Pilot Subcarriers

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Abstract

The use of pilot carriers to estimate a slowly changing frequency selective fading channel is investigated. The performance of a (bit and subcarrier adaptive) OFDM system over a 6 tap Rayleigh fading channel modeled by a 4 tap channel estimate is simulated. An algorithm is proposed for adaptive channel estimation using LMS.

Model

The attenuation of a high frequency wireless channels along a particular, (speficically not direct "line-of-sight") path is well modeled in magnitude (given standard assumptions (1)) by a Rayleigh random variable. To robustly simulate a wideband, frequency-selective fading channel, a 6-tap impulse response is generated, exponentially decreasing in power, where the magnitude of each tap is Rayleigh distributed, and the phase is uniform on 2π. We assume that the channel is constant within a single OFDM symbol time, but changes from symbol to symbol. Furthermore, because what is desired is a series of correlated channels, the random phases are held constant throughout a given simulation iteration, and a series of correlated random variables is generated for each tap's magnitude, where the autocorrelation of each tap is characterized by a given Doppler frequency, and the taps are independent of each other. Furthermore, the channel impulse response is scaled to produce an expected gain of one in each subcarrier. Thus, by using unity noise power in each subcarrier, average SNR can be easily varied by varying the transmit power. The efficient method used for the generation of these variables is given in (2). Figure 1 (stolen from (3)) shows a few symbol periods of a channel so generated.

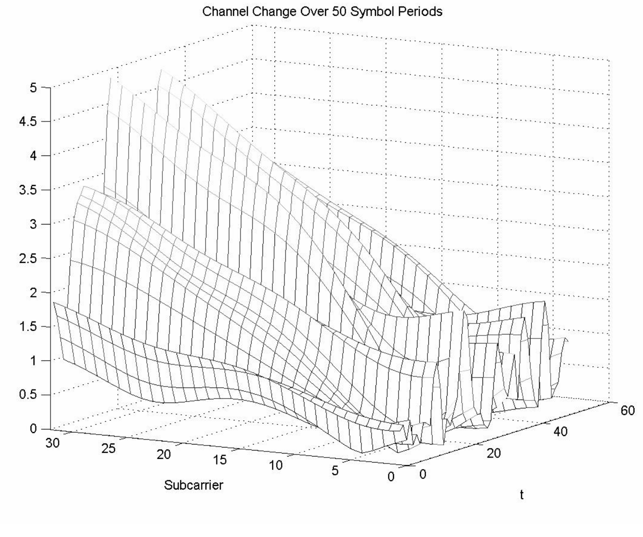

Figure 1: Example Channel Model

One of the advantages of an OFDM communication system is that it is very simple to "waterfill", the process through which power is optimally allocated across a frequency-selective fading channel. The intuition behind this idea is very simple. Rather than transmitting with a lot of power in frequency bins with low channel gains (to make up for the attenuation of the channel), it's better to transmit with less power, and instead to transmit with more power (and hence transmit more bits) in the frequency bins with high channel gains. The system used for this paper is not fully optimal, but takes some advantage of this idea. A maximum amount of power is set aside for each frequency bin, and the number of bits and transmit power are jointly set to achieve a P_b of 10^{-6} (using the tables in (4)), and to maximize the number of bits transmitted.

Channel Estimation Method

For the purposes of this paper, a simple channel estimation scheme was used. A fraction of the subcarriers were set aside as "pilots". We assume that the channel estimate for these pilots (in which no data is transmitted, but rather some control signal) is perfect. Then, for each pilot, the channel output at that frequency is generated by the dot product of the channel impulse response and the appropriate row of the DFT matrix. Thus, LMS can be used to find an appropriate impulse response (which was limited to 4 taps, purposefully less than the channel impulse response for realism (and also because it worked better)). The channel estimate was updated using each pilot, with a LMS update constant (μ) of 0.01 and that sequence was repeated for 50 iterations. As a result of the limited estimated channel impulse response, it is clear that an error floor is reached. As discussed in (5), the pilots were iterated circularly, so that a pilot occurred eventually in each subcarrier.

Results

The results are shown in Figures 2 and 3. Figure 2 shows the relationship between bit-error rate (BER) and the number of pilots in the OFDM system. For this simulation, 256 correlated channels with 64 subcarriers in each channel were generated for each simulation iteration, and 10 iterations were performed for each pilot configuration. The

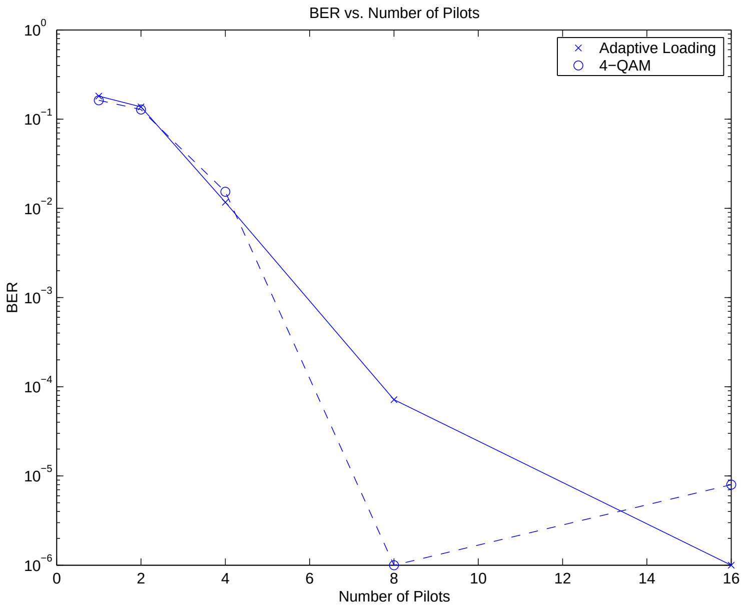

Figure 2: BER vs. Number of Pilots

sampled Doppler frequency was set to 0.05. The average SNR in each subcarrier is scaled to be 16 dB. As expected, the BER performance increases with the number of pilots. Notice the two lines in the figure. The second, labeled "4-QAM", refers to a situation in which the number of bits per subcarrier is limited to be either zero or two. By enforcing the power constraint, but limiting the maximum number of bits to two, a typical non-adaptive system with some cutoff-power is simulated. The performance is essentially the same, the artifact with 16 pilots is statistically insignificant. Also significant to note is that (though this is not depicted) even without channel estimation, the channel does not change enough to entirely destroy communications, but rather allows it at a BER of about 15%. This perhaps suggests a failure of the model.

Figure 3 shows the relationship between BER and the Doppler frequency. Notice that the Doppler frequencies depicted are as a fraction of the sampling rate, so that a Doppler frequency of, for example, 0.1, corresponds to 10% of the sampling frequency, which for a 250 kHz bandwidth system would be 250 kHz/32 samples per OFDM symbol, or a real Doppler frequency of 390 Hz. In this simulation, 512 correlated channels were used, with 32 subcarriers per OFDM symbol, and two pilot subcarriers. What is most delightful is how well the pilot-aided channel estimate system works at low Doppler frequencies.

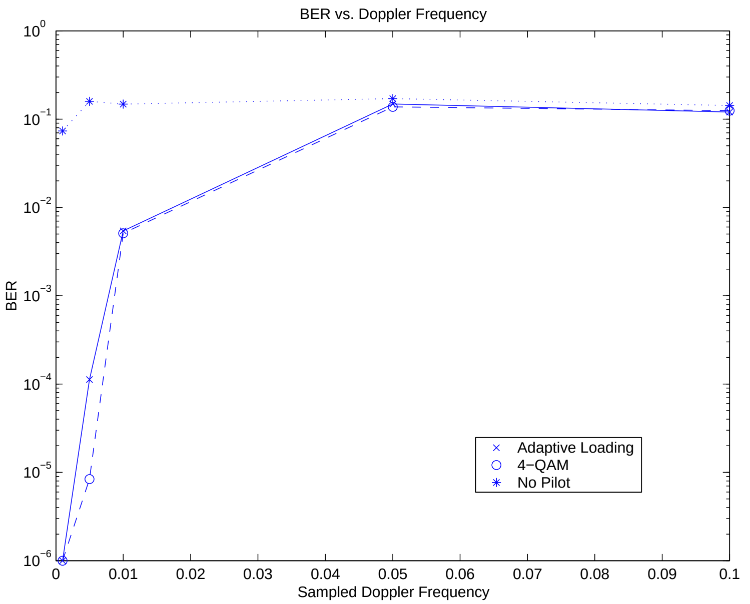

Figure 3: BER vs. Doppler Frequency

Further investigation would compare these results with the MMSE estimate generated in (5), and also with decision-aided channel estimates, which are also shown there to increase performance.

References

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3. C. Kemere, "Wideband Multiuser OFDM: Implementation and Capacity," EE360 Course Project, June 2001, Stanford University.
4. J. Cioffi, *EE379a Course Reader*, Stanford University, 2001.
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Appendix: Source Code:

```
ChanLength = 1024;
dopvec = [0.001, 0.005, 0.01, 0.05, 0.1];
Pmax = 40;
N = 32;

N_pilots = 2;
p = 0:ceil(N/N_pilots):(N-1);
mu = 0.01;
N_iter = 50;
for m = 1:N
    t = zeros(N,1);
    t(m) = 1;
    D(:,m) = fft(t);
end


% First, let's make a table of required QAM constellation energies:
SNRreq = [0, 13.7, 13.7, 18.5, 20.7, 23.7, 27.0, 29.8, 33.0, 35.8, 39.0,1000];
%SNRreq = [0, 13.7, 13.7, 1000];
Preq = 10.^(SNRreq/10);
Preq(1) = 0;
dPreq = diff(10.^(SNRreq/10));
Preq(1) = 1;

Pcons(1) = 1;
for n = 2:length(SNRreq)
    [t1,t2] = qaskenco(2^(n-1));
    Pcons(n) = var(t1+j*t2,1);
end
Preqcons = Preq./Pcons;

expProfile =  [1.0000, 0.7943, 0.1259, 0.1000, 0.0316, 0.0100];

sf = sum(expProfile.^2);
for dop = 1:length(dopvec)
    Doppler = dopvec(dop);
    for k = 1:10
        % Generate channel
        for m = 1:length(expProfile)
            H(m,:) = ray(ChanLength,Doppler,1).*exp(rand(1)*j*2*pi);
            H(m,:) = expProfile(m).*H(m,:);
            H(m,:) = H(m,:)./sqrt(2*sf);
        end
        Hf = fft(H,N); %(64x1024)

        % Encode Data
        c = zeros(N,ChanLength);
        h_est(1:4,1) = H(1:4,1);
        h = [h_est; zeros(N-length(h_est),1)];
        Hest = fft(h,N);

        tic
        for m = 1:ChanLength
             p = mod(p+1,N-1);

            % For each carrier, determine number of bits to transmit
            %  and generate both data and symbol for it
            flag = 0;
            for n = setdiff(1:N,p+1)
                % Determine bits
                while ( cumsum( (dPreq(1:c(n,m)+1))/(abs(Hf(n,m)).^2) ) < Pmax )
                    c(n,m) = c(n,m) + 1;
                end

                % Generate data and symbol
                if (c(n,m) == 0)
                    d(n,m) = 0;
                    x(n,m) = 0;
                    chan_x(n,m) = x(n,m)*Hf(n,m);
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                else
                    d(n,m) = randint(1,1,2^c(n,m));
                    [t1,t2] = qaskenco(d(n,m),2^c(n,m));
                    x(n,m) = (t1 + j*t2) * sqrt(Preqcons(c(n,m)+1)) / abs(Hf(n,m));
                    chan_x(n,m) = x(n,m)*Hf(n,m);
                end
            end

            %if (sum(c(:,m)) == 0)
            %fprintf('No bits transmitted.\n');
            %else
            %fprintf('%d bits transmitted.\n',sum(c(:,m)));
            %end

            % Do channel estimation (assume pilot recovery works)
            %for q = 1:N_iter
                %for r = 1:N_pilots
                    %err = Hf(p(r)+1,m) - h.'*D(:,p(r)+1);
                    %temp = conj(conj(h) + mu*D(:,p(r)+1)*err');
                    %h(1:4) = temp(1:4);
                    %end
                %end
            ch_err(dop,k,m) = sum(abs(fft(h,N) - Hf(:,m)).^2);
            %Hest = fft(h,N);

            % Generate and add AWGN
            noise = wgn(N,1,1,1,'linear','complex');
            y(:,m) = chan_x(:,m) + noise;

            % Decode channel output
            y_sc = y(:,m) ./ Hest;
            y_sc = y_sc .* (abs(Hf(:,m))./sqrt(Preqcons(c(:,m)+1))');
            for n = setdiff(1:N,p+1)
                if (c(n,m) > 0)
                    dy(n,m) = qaskdeco(real(y_sc(n)),imag(y_sc(n)),2^c(n,m));
                    % find number of bit errors
                    berr(n,m) = sum( abs(de2bi(d(n,m),c(n,m)) - de2bi(dy(n,m),c(n,m))) );
                else
                    dy(n,m) = 0;
                end
            end
            if (mod(m,50) == 1)
                fprintf('m = %d, k = %d\n',m,k);
            end
        end
        toc
        bits(k,dop) = sum(sum(c));
        errs(k,dop) = sum(sum(berr));
        BER(k,dop) = sum(sum(berr))/sum(sum(c))
    end
end
```